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Wave Propagation Approach to Fluid Filled Submerged Visco-Elastic Finite Cylindrical Shells

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Multi-layer orthotropic finite cylindrical shells with a viscoelastic core in contact with fluids are gaining increasing importance in engineering. Vibrational control of these structures is essential at higher modes. In this study, an extended version of the wave propagation approach using first-order shear deformation theory of shell motion is employed to examine the free vibration of damped finite cylindrical shells in vacuum or in contact with interior or exterior dense acoustic media. For this purpose, a one-layered viscoelastic finite cylindrical shell and a three-layered orthotropic finite cylindrical shell with a viscoelastic core layer were used. Complex natural frequencies have been extracted and the effects of fluid coupling on real and imaginary parts of natural frequencies have been examined. The results reveal that the fluid reduces the imaginary part as much as the real part of the damped natural frequency but that the proportion of the imaginary to the real part (loss factor) remains rather unchanged. Another aspect of the study involves the investigation of the effect of shell parameter, m, when the circumferential mode number, n, increases on both entities of damped natural frequencies. It is found that by increasing n, the real part of the natural frequency follows a u-shape trend; however, the imaginary part reduces and levels off for higher circumferential numbers. The loss factors remain almost constant for these higher modes. The results of the current approach are finally compared with ABAQUS solutions showing superiority of current approach.

INTRODUCTION

Cylindrical shells filled with fluid and/or submerged in acoustic media are the practical element of many types of engineering structures such as marine craft and airplanes in which machinery-induced vibration often occurs. Sound radiation from such shells still forms an important area of research in noise control. To suppress mechanical vibration and noise, a number of techniques can be used [1]. The amount of damping is the most important parameter in order to reduce the magnitude of response near resonant frequencies. If the resonance is inevitable, as in most cases, viscoelastic materials that have high loss factors are added to the structure to increase damping. The viscoelastic damping layer can passively control the response of the structure at a low cost but with a high reliability.

A structure vibrating in contact with a fluid of comparable density experiences radiation loading comparable to the elastic forces. The radiation loading, thus, modifies the forces acting on the structure, causing a "feedback coupling" to appear between the fluid and the structure. Thus, it is impossible to neglect the coupling between the structure and the fluid. Junger conducted pioneering studies of the vibrations of elastic spherical and cylindrical shells freely suspended in acoustic media to obtain damped complex natural frequencies [2]. Also, he presented a physical interpretation of the expression for an outgoing wave in the cylindrical co-ordinate and suggested practical applications of these phenomena [3]. Free and forced vibration problems of infinitely long thin cylindrical shells were considered by Bleich et

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al [4]. Warburton compared the natural frequencies of an infinitely long cylindrical shell in vacuum and in contact with acoustic media [5]. Later, Junger reviewed the bulk of studies in this field [6]. Numerous numerical methods such as the finite element method have since been used to study fluid-solid interaction phenomena. However, it is difficult to use analytical methods to take into account the exact effect of fluid-solid interactions, except in a few cases (e.g., spherical or infinite cylindrical structures). On the other hand, in cases where the submerged cylindrical shell is finite, the coupling problem is augmented because of the boundary conditions for the shell and unavailability of analytical formulae for the fluid. The wave propagation approach had been used by many researchers before Zhang et al [7-10] developed it to investigate coupled and uncoupled natural frequencies of different thin finite cylindrical shells. Since then, a number studies have shown its applications [11]. Recently Liu et al [12] investigated its application to analyze vibration characteristics of a buried pipe. Zhang et al [13] again used the wave propagation approach along with first order shear deformation theory [14-15] to calculate the natural frequency of orthotropic arterial wall without including damping properties of the shell.

Vibration and damping characteristics of cylindrical shells including viscoelastic materials have been investigated by numerous researchers using many different analytical and/or numerical approaches. Perhaps Edward [16] was the one to start investigating damped flexural waves. Axisymmetric vibrations of a finite-length cylindrical sandwich shell with a viscoelastic core layer have been studied by Pan [17]. Markus [18] studied the damping of cylindrical shells coated with unconstrained viscoelastic materials. Ramesh et al [19] studied natural frequencies and loss factors in cylindrical shells with a constrained viscoelastic layer between two isotropic, elastic facings using the finite element method. Sivadas [20] used a finite element method to study the free vibration and damping analysis of moderately thick cylindrical shells. At the same time, Ramesh et al [21] solved the problem of the vibration of a damped orthotropic cylindrical shell using the finite element method. Later, Ramasamy et al [22] employed the finite element method and calculated the natural frequencies and loss factors of fluid-filled vertical cylindrical shells with a constrained viscoelastic layer between two facings made of a composite material. Saravanan et al [23] solved a similar problem using the modal strain energy method. Also, Ganesan [24] et al studied a vertical fluid-filled and submerged cylindrical shell in which they treated the fluid domain using the Bessel function approach but the shell domain based on the shear deformation theory. In 2008, Hasheminejad et al [25] studied an infinitely long viscoelastic cylindrical shell in contact with internal and external acoustic fluids. Due to the unavailability of numerical results on the natural frequencies of viscoelatic submerged cylindrical shells, they verified their results against those of a steel shell under the conditions obtained by Warburton [5]. They assumed the loss modulus of steel to be equal to zero and verified their Matlab code. Recently, Hamidzadeh [26] has investigated the effect of the viscoelastic core thickness on the modal loss factors of a thick three-layered cylinder in vacuum.

In this study, the first order shear deformation theory of shell motions has been used to investigate the dynamic behavior (natural frequencies and loss factors) of cylindrical shells including the isotropic viscoelastic cylindrical shell as well as the three-layered orthotropic cylindrical shells with a viscoelastic core layer in contact with dense acoustic medium. The wave propagation approach has been employed as it can easily cope with different boundary conditions with different axial wave numbers [6]. In this approach, the natural mode of the cylindrical shell is treated as a combination of standing waves in the circumferential and axial directions. The circumferential standing wave is determined by its circumferential modal parameter, n, and the axial standing wave is determined by its axial modal parameter, m. Thus, the relation between the natural frequency and the standing wave parameters, n and m, is obtained. The axial wave number of the standing wave is approximately determined by the wave number of an equivalent beam with similar boundary conditions as the shell. In case the shell is elastic, the frequency equation is a complex transcendental equation with real coefficients and the imaginary part of the frequency goes to zero while a shell containing viscoelastic materials has a complex velocity of dilatation wave [26] and the coefficients of frequency equation are complex. Therefore, the natural frequencies obtained are no more real and solving this type of equation is rather more complicated. The results of the wave propagation approach have been verified against those obtained from the finite element ABAQUS code. Finally, the coupling effects on real and imaginary parts of the damped natural frequencies have been examined.

The wave propagation approach has been used for the study of neither the free vibrations of submerged fluid-filled viscoelastic cylindrical shells nor those of multilayered ones. All previous studies have employed other methods like FEM [22,23]. Also, published tabulated results for real and imaginary parts of higher modes are rare; for instance, in a recent study by Hasheminejad *et.al.* [25], they did not compare their results with a viscoelastic shell. The present study provides the detail of the real and imaginary parts of not only viscoelastic shells in vacuum but those of a submerged or fluid-filled one. In other studies including Hamidzadeh [26], the details of real and imaginary parts of natural frequencies associated with cylindrical shells in vacuum have been tabulated. However, he used a different method that failed to include end boundary conditions. This study was, therefore, carried out and high circumferential and longitudinal modes were calculated and compared with those obtained by ABAQUS to remedy the lack of elaborate results on viscoelastic shells in contact with a fluid and to enhance the current scant understanding of the imaginary part of the natural frequency. A final outcome of the study is the demonstration of the accuracy of the wave propagation approach.

FORMULATION

The geometry of the shell and coordinate system are shown in Figure 1. The cylindrical shell is assumed to have length L, thickness h, and radius R. The orthogonal co-ordinate system (x, θ, z) is fixed at the mid surface of the cylindrical shell in which x is the axial direction, θ is the circumferential direction and z is the radial direction. The deformations of the shell are defined by u, v, w, ϕ_x and ϕ_θ which are displacements of the point in (x, θ, z) and the rotation of the trasverse normal about θ and x axis are respectively:

$$\frac{\partial u}{\partial z} = \phi_x$$
$$\frac{\partial v}{\partial z} = \phi_\theta \qquad (1)$$

First order theory is used to deal with the influence of shear forces on the frequencies of the thick shell. In the first order shear deformation theory, the Kirchhoff

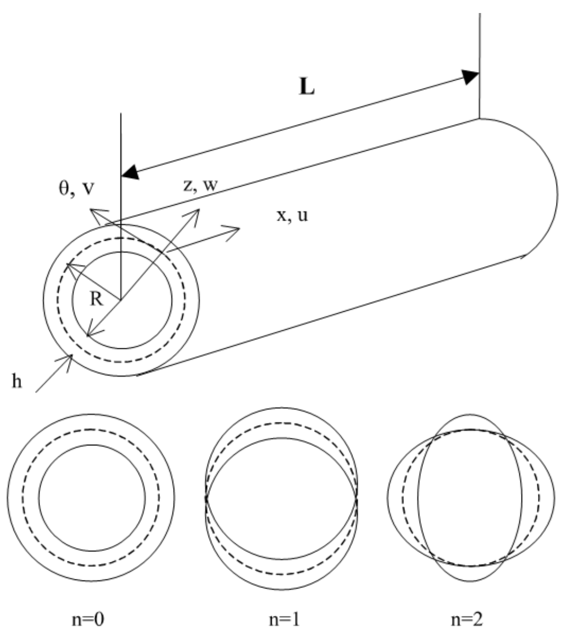

Figure 1. Co-ordinate system and circumferential modal shapes.

hypothesis is relaxed by not constraining the transverse normals to remain perpendicular to the mid surface after deformation. According to the theory the displacement fields are of the form:

$$u(x, \theta, z, t) = u_0(x, \theta, t) + z\phi_x(x, \theta, t)$$
$$v(x, \theta, z, t) = v_0(x, \theta, t) + z\phi_\theta(x, \theta, t)$$
$$w(x, \theta, z, t) = w_0(x, \theta, t) \qquad (2)$$

where u_0, v_0, w_0, ϕ_x and ϕ_θ are unknowns to be determined and also u_0, v_0, w_0, ϕ_x and ϕ_θ are the displacements on the surface $z = 0$ and the rotations of transverse normal about its θ and x axis respectively.

The equations of free vibration with first order shear deformation theory (FSDT) of a cylindrical shell using Donnell's theory in terms of forces and moment resultants:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} - I_0\frac{\partial^2 u}{\partial t^2} - I_1\frac{\partial^2 \phi_x}{\partial t^2} = 0$$
$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial \theta} - I_0\frac{\partial^2 v}{\partial t^2} - I_1\frac{\partial^2 \phi_\theta}{\partial t^2} = 0$$
$$\frac{\partial Q_x}{\partial x} + \frac{1}{R}\frac{\partial Q_\theta}{\partial \theta} - \frac{N_\theta}{R} - I_0\frac{\partial^2 w}{\partial t^2} = 0$$
$$\frac{\partial M_x}{\partial x} + \frac{1}{R}\frac{\partial M_{\theta x}}{\partial \theta} - Q_x - I_1\frac{\partial^2 u}{\partial t^2} - I_2\frac{\partial^2 \phi_x}{\partial t^2} = 0$$
$$\frac{\partial M_{\theta x}}{\partial x} + \frac{1}{R}\frac{\partial M_\theta}{\partial \theta} - Q_\theta - I_1\frac{\partial^2 v}{\partial t^2} - I_2\frac{\partial^2 \phi_\theta}{\partial t^2} = 0 \qquad (3)$$

where N and M are the force and moment resultants defined as:

$$\{N_x, N_\theta, N_{x\theta}, N_{\theta x}\} =$$
$$\sum_{k=1}^{N}\int_{z_{k+1}}^{z_k}\{\sigma_x(1+z/R), \sigma_\theta, \sigma_{x\theta}(1+z/R), \sigma_{x\theta}\}dz$$
$$\{M_x, M_\theta, M_{x\theta}, M_{\theta x}\} =$$
$$\sum_{k=1}^{N}\int_{z_{k+1}}^{z_k}\{\sigma_x(1+z/R), \sigma_\theta, \sigma_{x\theta}(1+z/R), \sigma_{x\theta}\}zdz$$

$$\{Q_x, Q_\theta\} = \sum_{k=1}^{N}\int_{z_{k+1}}^{z_k}\{\sigma_{xz}(1+z/R), \sigma_{\theta z}\}\,zdz \qquad (4)$$

and the mass moments of inertia are:

$$\{I_0, I_1, I_2\} = \sum_{k=1}^{N}\int_{z_{k+1}}^{z_k}\rho^k(1, z, z^2)dz \qquad (5)$$

the terms z/R in Eqs. (4) and (5) when considering thick shells are so large which cannot be neglected.

For a cylindrical shell, stresses defined in Eq. (4) are defined by two dimensional Hooke's law as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{\theta z} \\ \sigma_{xz} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} e_x \\ e_\theta \\ e_{\theta z} \\ e_{xz} \\ e_{x\theta} \end{Bmatrix} \tag{6}$$

The strain components in Eq. (6) are defined by:

$$\{e_x, e_\theta, e_{\theta z}, e_{xz}, e_{x\theta}\} =$$

$$\left\{\frac{\partial u}{\partial x}, \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right), \frac{\partial v}{\partial z} + \frac{1}{R}\frac{\partial w}{\partial \theta}, \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right\} \tag{7}$$

which can be written separately with first order shear deformation theory as:

$$\begin{Bmatrix} e_x \\ e_\theta \\ e_{\theta z} \\ e_{xz} \\ e_{x\theta} \end{Bmatrix} = \begin{Bmatrix} e_1 \\ e_2 \\ e_4 \\ e_5 \\ e_6 \end{Bmatrix} + z \begin{Bmatrix} \kappa_1 \\ \kappa_2 \\ 0 \\ 0 \\ \kappa_6 \end{Bmatrix} \tag{8}$$

where

$$\{e_1, e_2, e_4, e_5, e_6\} = \left\{\frac{\partial u_0}{\partial x}, \frac{1}{R}\left(\frac{\partial v_0}{\partial \theta} + w_0\right), \phi_\theta + \frac{1}{R}\frac{\partial w_0}{\partial \theta},\right.$$

$$\left.\phi_x + \frac{\partial w_0}{\partial x}, \frac{\partial v_0}{\partial x} + \frac{1}{R}\frac{\partial u_0}{\partial \theta}\right\}$$

and

$$\{\kappa_1, \kappa_2, \kappa_6\} = \left\{\frac{\partial \phi_x}{\partial x}, \frac{1}{R}\frac{\partial \phi_\theta}{\partial \theta}, \frac{\partial \phi_\theta}{\partial x} + \frac{1}{R}\frac{\partial \phi_x}{\partial \theta}\right\}$$

By substituting Eqs. (7) and (8) into Eq. (6) and then substituting the resulting equation into Eq. (4) , the force and moment resultants can be derived as:

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ N_{\theta x} \\ M_x \\ M_\theta \\ M_{x\theta} \\ M_{\theta x} \\ Q_x \\ Q_\theta \end{Bmatrix} = s \begin{Bmatrix} e_1 \\ e_2 \\ e_4 \\ e_5 \\ e_6 \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{Bmatrix}$$

$$s = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & a & b & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{66} & 0 & 0 & c \\ 0 & 0 & 0 & 0 & A_{66} & 0 & 0 & 0 \\ a & b & 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & c & 0 & 0 & D_{66} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{66} \\ 0 & 0 & 0 & KA_{55} & 0 & 0 & 0 & 0 \\ 0 & 0 & KA_{44} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$a = D_{11}/R,\ b = D_{12}/R,\ c = D_{66}/R \tag{9}$$

where K is the shear correction factor [13,14,15] and details are not mentioned here for the sake of brevity, also A_{ij}, D_{ij} are the extensional and bending stiffness and when the shell is just one layer they are defined respectively as:

$$(A_{ij}, D_{ij}) = \sum_{k=1}^{k=N} \int_{z_{k+1}}^{z_k} Q_{ij}(1, z^2) dz \tag{10}$$

For an anisotropic shell, which is assumed to be in a state of plane stress, Q_{ij} are the reduced stiffness defined as:

$$Q_{11} = \frac{E_{11}}{1 - \upsilon_{12}\upsilon_{21}}, Q_{12} = \frac{\upsilon_{12}E_{22}}{1 - \upsilon_{12}\upsilon_{21}}, Q_{11} = \frac{E_{22}}{1 - \upsilon_{12}\upsilon_{21}}$$

$$Q_{44} = G_{23}, Q_{55} = G_{13}, Q_{66} = G_{12} \tag{11}$$

where E_{11} and E_{22} are the Young's moduli, G_{12}, G_{13} and G_{23} are the shear moduli, ν_{12} and ν_{21} are the poison's ratios satisfying $\nu_{21}E_{11} = \nu_{12}E_{22}$.

Composite laminated shells have several layers with different orientations which may not coincide with global co-ordinate system used in the solution of the problem. In this case we need to establish the transformation relation:

$$[\bar{Q}] = [T]\,[Q]\,[T]^T \tag{12}$$

where

$$[T]=\begin{bmatrix} \cos^2\theta & \sin^2\theta & 0 & 0 & -2B^* \\ \sin^2\theta & \cos^2\theta & 0 & 0 & 2B^* \\ 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & -\sin\theta & \cos\theta & 0 \\ \sin\theta & -B^* & 0 & 0 & \cos^2\theta-\sin^2\theta \end{bmatrix}$$

$$B^* = \sin\theta\,\cos\theta$$

In the case of composite shell with different orientations in each layer, Q_{ij} in Eq. (10) will be replaced by $\bar{Q}_{ij}$ obtained by Eq. (13) in this study the facing layers have no rotation or $\theta = 0$.

When considering an isotropic layer, $E_{11} = E_{22}$ and shear constants will be defined as [13]:

$$G_{12} = G_{13} = \frac{E_{11}}{2(1+\upsilon_{12})}\,,\; G_{23} = \frac{E_{22}}{2(1+\upsilon_{21})} \tag{13}$$

in which $\nu_{21} = \nu_{12} = \nu$. It is obvious that when the number of layers reduces to one, Z_{k+1} and Z_k will be replaced by $-h/2$ and $h/2$ respectively.

By substituting Eq. (9), with substituting from Eqs. (10) and (11) into Eq. (3), yields five equations of motion in matrix form, namely,

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \phi_x \\ \phi_\theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{14}$$

where $L_{ij}(i,j = 1,5)$ are the differential operators [13-15].

The fluid in contact with cylindrical shell is assumed non-viscous and isotropic which satisfies the acoustic wave equation. The equation of motion of the fluid can be written in the cylindrical co-ordinate system (x,θ,r) as [8, 10]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{15}$$

Where p is the acoustic pressure and c is the sound speed of the fluid. The x and θ co-ordinates are the same as those of the shell, where the r co-ordinate is taken from the x-axis of the shell.

WAVE PROPAGATION APPROACH

In the wave propagation approach the displacements of the shell are expressed in the format of wave propagation, associated with the longitudinal wave number κ_x and the circumferential modal parameter n and defined by:

$$u_0 = U_m\,\cos(n\theta)\,e^{(i\omega t - ik_x x)}$$

$$v_0 = V_m\,\sin(n\theta)\,e^{(i\omega t - ik_x x)}$$

$$w_0 = W_m\,\cos(n\theta)\,e^{(i\omega t - ik_x x)}$$

$$\phi_x = (\Phi_{xm}/R)\,\cos(n\theta)\,e^{(i\omega t - ik_x x)}$$

$$\phi_\theta = (\Phi_{\theta m}/R)\,\sin(n\theta)\,e^{(i\omega t - ik_x x)} \tag{16}$$

where U_m, V_m and W_m are respectively, the displacement amplitudes in x,θ,z directions, Φ_{xm} and $\Phi_{\theta m}$, respectively, the rotation amplitudes of transverse normal about θ and x axis, ω is circular frequency.

The associated form of the acoustic pressure filed in the interior fluid, which satisfies the acoustic wave Eq. (15), can be expressed in the cylindrical co-ordinate system, associated with an axial wave number κ_x, radial wave number κ_r and circumferential modal parameter n, and given as:

$$p = P_m \cos(n\theta)\, J_n(k_r r) e^{(i\omega t - ik_x x)} \tag{17}$$

where $J_n()$ is the Bessel function of order n. This function is replace with second kind Hankel function of order n when considering exterior acoustic medium, then the acoustic pressure satisfying wave Eq. (16) is given as:

$$p = P_m \cos(n\theta)\, H_n^{(2)}(k_r r) e^{(i\omega t - ik_x x)} \tag{18}$$

where P_m is the pressure amplitude, $H_n^{(2)}()$ is the second kind Hankel function. The radial wave number κ_r is related to axial wave number κ_x by the usual vector relation $\kappa_r^2 + \kappa_x^2 = \kappa_0^2$ where $\kappa_0 = \omega/c$ is the fluid acoustic wave number.

To ensure that fluid remains in contact with shell wall, the fluid radial displacement and the shell radial displacement must be equal both at the interface of shell with outer fluid and inner one [8,10]. This coupling condition is then:

$$-\{1/i\omega\rho_f\}\,(\partial p/\partial r)|_{r=R} = (\partial w_0/\partial t)|_{r=R} \tag{19}$$

which for exterior fluid gives [10]:

$$P_m = \left[\omega^2 \rho_f \Big/ k_r H_n^{'(2)}(k_r R)\right] W_m \tag{20}$$

where ρ_f is the density of fluid and the prime on the $H_n^{(2)}()$ denotes differentiation with respect to the argument $\kappa_r R$.

Substituting Eqs. (16), (17) or (18) into Eqs. (14), with consideration of coupling Eq. (20), results in the equation of motion of coupled system in matrix form as:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \end{bmatrix} \begin{Bmatrix} U_m \\ V_m \\ W_m \\ \Phi_{xm} \\ \Phi_{\theta m} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

Expansion of the determinant of the amplitude coefficient in Eq. (21) provides the dispersion equation in the form

$$D(k_x, \omega) = 0 \tag{22}$$

Axial wave number is determined based on two boundary conditions [8] and in this study just clamped-clamped boundary condition is examined for the sake of brevity. Then the dispersion Eq. (22) can be written as

$$P_1(\omega) + P_2(\omega)\,FL = 0 \tag{23}$$

where $P_1(\omega)$ and $P_2(\omega)$ are polynomial functions of ω and FL is the term of fluid loading effect which for a submerged cylindrical shell [10] is in the form of:

$$FL = -\frac{\rho_f \omega^2}{k_r} \frac{H_n^{(2)}(k_r R)}{H_n^{'(2)}(k_r R)} \tag{24}$$

and for a fluid filled cylindrical shell [8] can be in the form:

$$FL = \frac{\rho_f \omega^2}{k_r} \frac{J_n(k_r R)}{J_n'(k_r R)} \tag{25}$$

And, for a submerged fluid-filled cylindrical shell, both of these fluid loading effects are accumulated. In case the shell motion is investigated using a classical thin shell theory such as Love's theory [10], $P_1(\omega)$ is a polynomial to a maximum power of six and $P_2(\omega)$ is a polynomial to a maximum power of four. However, these powers change to ten and eight, respectively, when the first order shear deformation is used. Considering an elastic undamped shell leads to real coefficients of these polynomials, while a viscoelastic shell can result in complex coefficients. On the other hand, adding a complex function like Hankle's function whose argument depends on ω, it results in a complex transcendental equation which needs a sophisticated computational procedure to be solved. Damped natural frequencies have been obtained in a complex form and the proportion of the imaginary part f_I to the real part f_R is defined as the loss factor associated with a specific modal shape.

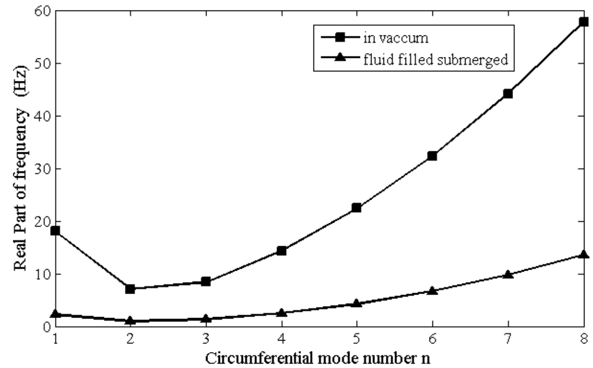

Figure 2. Comparison of coupled and uncoupled real part of frequency ($m = 1$) for a viscoelastic shell, because of fluid added mass the variation of real part remains similar to that is discussed in [10] for a elastic shell.

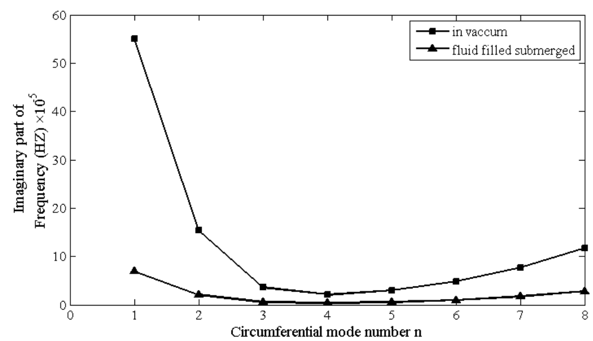

Figure 3. Comparison of coupled and uncoupled imaginary part of frequency ($m = 1$) for a viscoelastic shell, fluid has a similar effect on imaginary part and variation is similar to that of real part.

NUMERICAL RESULTS AND DISCUSSION

The accuracy of the wave propagation approach for calculating coupled and uncoupled natural frequencies of isotropic cylindrical shell applying Love's shell motion theory have been previously validated in Refs. [7-10]. The focus of this paper is to validate the accuracy of the method for calculating complex natural frequencies of damped cylindrical shells including one-layered viscoelastic shells or three-layer orthotropic shells with a viscoelastic core layer. Also, the effects of interior or exterior dense acoustic media, or both, on the imaginary part of natural frequencies have been examined.

As few results have been reported in the literature on the damped natural frequencies of finite cylindrical shells in vacuum or in contact with dense acoustic media, the finite element ABAQUS code is used in this study to verify the accuracy of the method. The complex natural frequency extraction procedure [27] is used to obtain natural frequencies of a damped shell. ABAQUS offers the subspace projection method to solve for complex eigenvalues and right eigenvectors.

In this method, the damped eigensystem is projected onto a subspace spanned by the eigenvectors of the undamped system. Thus, the undamped eigenproblem must be solved prior to the complex eigenvalue extraction procedure. The ABAQUS model takes into account the mechanical properties of both the shell and the fluid. The loss moduli of the viscoelastic shell are defined using the viscoelatic tabular definition and the sound speed in an acoustic medium is defined through Bulk moduli for ABAQUS.

A model of a finite cylindrical shell with end-caps like in Refs. [10] was created. The length and radius of shell are, respectively, 20 and 1 meter. In case the shell is one layer the thickness is assumed 1 centimeter while in a three layer one the viscoelastic thickness is 1 millimeter and both of constraining layers are 4 millimeter thick. All shells are fully clamped at both ends and inner or outer fluid in all cases are water with density of $\rho_f = 1000\,kg/m^3$ and sound speed of $c = 1500\,m/s$.

Table 1 shows complex natural frequencies of a viscoelastic cylindrical shell in vacuum. The storage and loss moduli of shell are $2.3 \times 10^7 N/m^2$ and $0.8 \times 10^7 N/m^2$ respectively with poison ratio of $\nu = 0.34$ and density of $\rho = 1340 kg/m^3$. The real part and imaginary part of natural frequencies related to eight modal shapes calculated with the wave propagation approach have been compared with those of ABAQUS finite element software (FEM). The modal shapes can be regarded as the mode (m, n), where m is the modal number in the axial direction and n is the modal number in the circumferential direction.

In order to compare the first order shear deformation theory in calculating coupled natural frequencies of a cylindrical shell and validating the accuracy of code written in Mathematica, the loss moduli is assumed to be zero and storage moduli is assumed to be $2.1 \times 10^{11} N/m^2$ and density equal to $7850 kg/m^3$ and poison ratio of $\nu = 0.3$ then the results can be compared with those of Loves's shell motion theory obtained in Refs. [8, 10] and ABAQUS and SYSNOISE [8, 10]. Table 2 and Table 3 compare the results of Love's theory and first order shear deformation theory in calculating coupled natural frequencies of a clamped-clamped cylindrical shell. According to Table 2, 3 the results obtained with first order shear deformation theory are higher than those of Love's theory and closer to those of ABAQUS and good agreement are seen.

In order to examine the effect of fluid on damped natural frequencies of a viscoelastic cylindrical shell interior and exterior fluid and both of them are added to problem and the real and imaginary parts of natural frequencies are tabulated in Tables 4-7. Adding fluid on one side of the shell reduces both entities of natural frequencies considerably because of the mass of fluid moving with shell but existence of fluid on both side

Table 1. Comparison of complex natural frequencies of a clamped/clamped viscoelastic cylindrical shell in vacu between ABAQUS and present method (wave propagation approach) ($L = 20$ m, $R = 1$ m, $h = 1$ cm).

Frequency (Hz)					
Order	Modal shape	$f_R(ABAQUS)$	$f_R(present)$	$f_I(ABAQUS)$	$f_I(present)$
1	(1,2)	0.308142	0.349599	0.05237775	0.0590644
2	(1,3)	0.526625	0.571253	0.08949925	0.0965125
3	(2,3)	0.613577	0.657048	0.1042765	0.111008
4	(2,2)	0.691889	0.739166	0.117608	0.124881
5	(3,3)	0.823599	0.866607	0.139973	0.146412
6	(1,4)	1.02032	1.01405	0.17335875	0.171323
7	(2,4)	1.04072	1.03624	0.17681975	0.175071
8	(3,4)	1.09676	1.09475	0.1863375	0.184957
9	(4,3)	1.15566	1.20338	0.196414	0.20331
10	(4,4)	1.209	1.21005	0.205406	0.204436

Table 2. Comparison of coupled natural frequencies of submerged isotropic cylindrical shell between results of Love's theory and first order shear deformation theory ($L = 20$ m, $R = 1$ m, $h = 1$ cm).

Frequency (Hz)					
Order	Modal shape	ABAQUS	SYSNOISE[10]	Love[10]	present
1	(1,2)	5.00	4.92	4.95	5.51
2	(1,3)	9.62	9.06	8.95	10.00
3	(2,3)	11.22	10.71	10.66	11.58
4	(2,2)	11.39	11.24	11.54	11.83
5	(3,3)	15.18	14.7	14.73	15.42
6	(1,4)	20.58	18.68	18.26	19.46
7	(2,4)	20.96	19.14	18.71	19.93
8	(3,4)	22.10	20.37	20.00	21.14

of the shell, although the mass has increased, does not have as significant effect as those in submerged or fluid filled conditions.

Viscoelastic shells are used to modify damping properties of shells through dissipating energy. So wave propagation approach is used again to calculate damped natural frequencies of a three layered cylindrical shell with constrained viscoelastic core layer. In this case the facing layers are assumed to be orthotropic like those in Refs. [13, 15] with $E_{11} = 19 \times 10^7 N/m^2$, other specifications are not mentioned here for the sake of brevity and viscoelastic layer is similar to previous stages. According to Tables 4-7 there are good agreement between the results of this method and those of finite element method and this method can calculate the complex natural frequencies of three layered damped cylindrical shells in vacuum or in contact with acoustic medium accurately.

For further investigation, the effect of fluid on real and imaginary parts of damped natural frequencies in different circumferential mode numbers is compared. Figure 2 shows the coupling effect on real parts of natural frequencies and Figure 3 illustrates this effect on imaginary parts. According to Figures 2 and 3, solving the problem of free vibration of a coupled damped cylindrical shell with the wave propagation approach predicts a similar effect on real and imaginary parts of damped natural frequencies and simulates their reduction because of added mass of fluid. The variation of real part of natural frequency in a damped shell is like an undamped one which is discussed in [10] surprisingly, imaginary part has a similar trend about which the difference between coupled and uncoupled imaginary parts first decreases and after passing the fundamental frequency this difference increases. On the other hand comparing the loss factors of a fluid filled submerged damped cylindrical shell with a similar shell in vacuum (Figure 4), shows that the fluid does not have a considerable effect on loss factors because the fluid has been assumed to be invicid and the

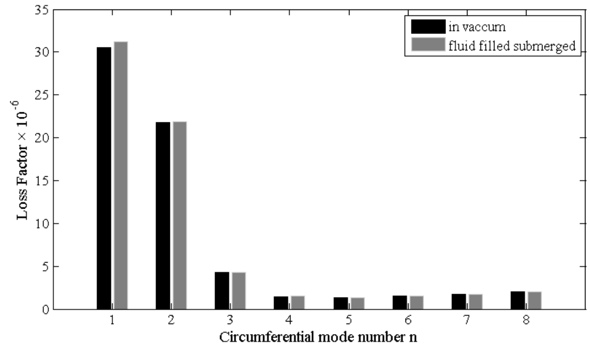

Figure 4. Comparison of coupled and uncoupled loss factors ($m = 1$), although both real and imaginary parts of natural frequency has changed the loss factor has slight changes because fluid has been assumed inviscid.

Table 3. Comparison of coupled natural frequencies of fluid filled isotropic cylindrical shell between results of Love's theory and first order shear deformation theory ($L = 20$ m, $R = 1$ m, $h = 1$ cm).

Frequency (Hz)					
Order	Modal shape	ABAQUS	SYSNOISE[8]	Love[8]	present
1	(1,2)	4.90	4.91	4.93	5.49
2	(1,3)	9.23	9.13	8.94	9.99
3	(2,3)	10.80	10.8	10.64	11.56
4	(2,2)	11.10	11.19	11.48	11.74
5	(3,3)	14.60	14.79	14.66	15.36
6	(1,4)	19.42	18.99	18.26	19.46
7	(2,4)	19.80	19.46	18.73	19.92
8	(3,4)	20.94	20.7	19.96	21.10

Table 4. Comparison of complex natural frequencies of a clamped/clamped fluid filled viscoelastic cylindrical shell between ABAQUS and present method (wave propagation approach) ($L = 20$ m, $R = 1$ m, $h = 1$ cm).

Frequency (Hz)					
Order	Modal shape	$f_R(ABAQUS)$	$f_R(present)$	$f_I(ABAQUS)$	$f_I(present)$
1	(1,2)	0.0552681	0.0632164	0.009394425	0.0106803
2	(1,3)	0.107356	0.118314	0.018245	0.019989
3	(2,2)	0.12584	0.136457	0.02138745	0.0230543
4	(2,3)	0.168524	0.180689	0.028641	0.0305272
5	(3,3)	0.229212	0.246527	0.0389615	0.0416504
6	(3,2)	0.230387	0.235592	0.03914375	0.039803
7	(1,4)	0.234889	0.241092	0.03990775	0.0407323
8	(4,3)	0.237055	0.252155	0.0402885	0.0426013
9	(2,4)	0.27075	0.282901	0.046013	0.0477959
10	(4,4)	0.313864	0.327109	0.0533265	0.0552647

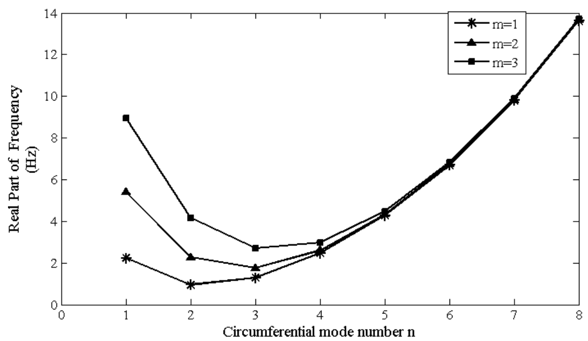

Figure 5. Variation of real part of natural frequency in different axial wave numbers when n is increasing for a fluid filled submerged cylindrical shell $L = 20$ m, $R = 1$ m, $h = 1$ cm, this variation is similar to that of an elastic shell.

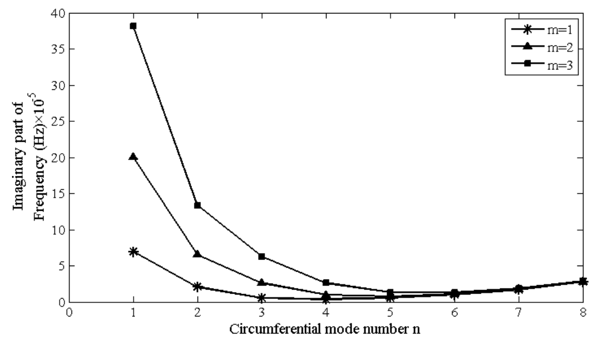

Figure 6. Variation of imaginary part of natural frequency in different axial wave numbers when n is increasing for a fluid filled submerged cylindrical shell $L = 20$ m, $R = 1$ m, $h = 1$ cm, this variations follow an unpredictable trend which is different from real part, and when n is high imaginary part remains almost unchanged.

damping effects have just been included through viscoelastic layer. In other words, although the imaginary and real parts of natural frequencies have changed, the proportion of these two entities remains rather constant.

It is interesting to investigate how axial mode m can change the real and imaginary parts of damped natural frequencies. Figure 5 shows the variation of real part of frequency for different axial modes when circumferential mode n is increasing. Similarly Figure 6 illustrates these changes related to imaginary part of natural frequencies. The variation of real part, similar to the results for an elastic shell [10], is u shape; however, the imaginary part of natural frequency follows a downward trend in all axial numbers and for higher circumferential mode numbers increases with lower rate and remains almost unchanged. Thus, according to Figure 7 loss factor in higher circumferential numbers

Table 5. Comparison of complex natural frequencies of a clamped/clamped submerged viscoelastic cylindrical shell between ABAQUS and present method (wave propagation approach) ($L = 20$ m, $R = 1$ m, $h = 1$ cm).

Frequency (Hz)					
Order	Modal shape	$f_R(ABAQUS)$	$f_R(present)$	$f_I(ABAQUS)$	$f_I(present)$
1	(1,2)	0.056837	0.0634786	0.0096611	0.0107246
2	(1,3)	0.113061	0.118443	0.01921595	0.0200108
3	(2,3)	0.128422	0.136025	0.02182785	0.0229812
4	(2,2)	0.13168	0.13686	0.02237815	0.0231223
5	(3,3)	0.176714	0.181697	0.03003075	0.0306975
6	(1,4)	0.237131	0.251144	0.040301	0.0424305
7	(2,4)	0.24794	0.235694	0.04212175	0.0398203
8	(3,4)	0.248556	0.254378	0.04223825	0.0429769
9	(4,3)	0.252156	0.24138	0.042837	0.0407809
10	(4,4)	0.29247	0.283952	0.04968275	0.0479734

Table 6. Comparison of complex natural frequencies of a clamped/clamped fluid filled submerged viscoelastic cylindrical shell between ABAQUS and present method (wave propagation approach) ($L = 20$ m, $R = 1$ m, $h = 1$ cm)

Frequency (Hz)					
Order	Modal shape	$f_R(ABAQUS)$	$f_R(present)$	$f_I(ABAQUS)$	$f_I(present)$
1	(1,2)	0.04006	0.045165	0.006809	0.007631
2	(1,3)	0.078671	0.08462	0.01337	0.014296
3	(2,2)	0.09013	0.096489	0.01532	0.016302
4	(2,3)	0.091935	0.097694	0.015624	0.016505
5	(3,3)	0.123357	0.129545	0.020964	0.021886
6	(3,2)	0.166348	0.177463	0.028275	0.029982
7	(1,4)	0.171208	0.168921	0.029089	0.028539
8	(4,3)	0.173565	0.181098	0.029497	0.030596
9	(2,4)	0.174369	0.172939	0.029625	0.029218
10	(3,4)	0.183782	0.183204	0.031223	0.030952

remains almost unchanged and lower circumferential numbers have higher loss factors. Surprisingly, comparing Figures 5, 6 and 7 shows that in higher circumferential numbers, although the denominator of the fraction which result in loss factor remains unchanged (Figure 6) and its numerator is increasing (Figure 5) , the loss factor does not change (Figure 7).

CONCLUSION

The article has extended wave propagation approach to investigate about damped natural frequencies of three layered orthotropic finite cylindrical shell with viscoelastic core layer in contact with acoustic medium. Several numerical examples have been solved using first order shear deformation theory and in case of a viscoelastic layer the problem has been simplified to an elastic shell to compare the results of Love's theory in predicting the coupled natural frequencies of an elastic finite cylindrical shell with those of with first order shear deformation theory. Also ABAQUS finite element code has been used to verify the complex natural frequencies obtained with present method and good agreement has been seen. Eight mode shapes have been extracted with complex natural frequency extraction procedure [27] in ABAQUS to verify the results of wave propagation approach to examine the accuracy of this method to calculate higher circumferential or axial mode numbers. This method can approximate higher modes accurately compare with ABAQUS results.

Also the variation of real and imaginary parts of damped natural frequencies associated with adding interior or exterior fluid or both of them have been considered. Effect of interior and exterior fluid reduces both real and imaginary part of natural frequencies considerably but effect of fluid on both side of the shell does not have a significant effect on natural frequencies. Also, although coupling effects changes the magnitude of damped natural frequencies, the loss factor remains almost unchanged and just added mass effect of fluid is notable in this type of simulation.

It is noticeable that real part of natural frequencies in different axial wave numbers follows a u shape model when circumferential mode number is increasing while the imaginary part has a downward trend and for higher circumferential mode numbers increases slightly; thus, the loss factor for higher circumferential mode numbers remains almost unchanged. Also the increase of real part natural frequency in higher circumferential mode numbers do not have dominant effect on loss facor and loss factor remains unchaned similar to the imaginary part of the natural frequency. s on free vibrations of a submerged fluid-filled thin cylindrical shell.

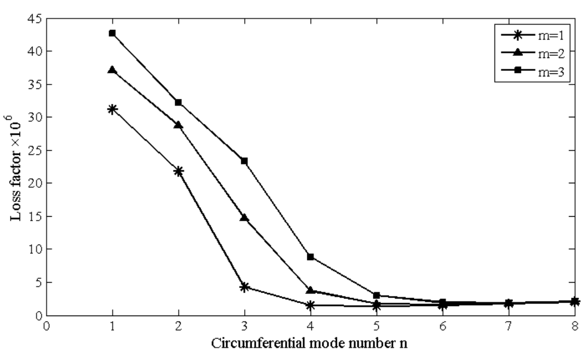

Figure 7. Variation of loss factor of natural frequency in different axial wave numbers when n is increasing for a fluid filled submerged cylindrical shell $L = 20$ m, $R = 1$ m, $h = 1$ cm , although for high n the real part is increasing and imaginary part is almost unchanged, the loss factor is unchanged.

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Table 7. Comparison of complex natural frequencies of a clamped/clamped three layer damped cylindrical shell in vacuum between ABAQUS and present method (wave propagation approach) ($L = 20$ m, $R = 1$ m, $h = 0.9$ cm).

Frequency (Hz)					
Order	Modal shape	$f_R(ABAQUS)$	$f_R(present)$	$f_I(ABAQUS)$	$f_I(present)$
1	(1,2)	6.54162	7.04315	1.59E-04	1.54E-04
2	(1,3)	7.67255	8.38106	3.60E-05	3.57E-05
3	(2,3)	10.6733	11.3385	1.70E-04	1.67E-04
4	(1,4)	13.8427	14.2942	1.61E-05	2.11E-05
5	(2,4)	14.5922	15.0628	5.13E-05	5.54E-05
6	(2,2)	15.7853	16.7851	4.71E-04	4.80E-04
7	(3,4)	16.6062	17.0919	1.47E-04	1.50E-04
8	(4,4)	20.2591	20.7733	3.12E-04	3.15E-04
9	(1,5)	22.6219	22.3939	1.84E-05	3.01E-05
10	(2,5)	22.8836	22.6896	2.81E-05	3.96E-05

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