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Keynote Contribution

A numerical study on slip flow heat transfer in Micro-Poiseuille flow

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Abstract
In the present study, two-dimensional incompressible momentum and energy equations are solved with slip velocity and temperature jump boundary conditions in a parallel plate micro channel. The computations are performed for micro channels with CHF (Constant Heat Flux) boundary conditions to obtain heat transfer characteristics of gaseous flow in slip regime. The effects of creep flow and viscous dissipation are neglected in this study. The numerical methodology is based on Semi-Implicit Method for Pressure-linked Equations (SIMPLE) method. the governing equations are developed by using perturbation expansions of velocity, pressure and temperature fields. It was found that Nusselt number and was substantially reduced for slip flow regimes compared with the continuum flows. The obtained solutions are compared with available numerical and analytical results and found that present study has good agreement with that works.

Keywords: heat transfer, slip-flow, micro-channel, parallel-plates, Nusselt number, perturbation expansion

1. Introduction

Research interest on flow and heat transfer phenomena in microchannels has increased substantially in recent years due to developments in the electronic industry, microfabrication technologies, biomedical engineering, etc. Fabrication of small devices has increased the needs of understanding of fluid flow and heat transfer in micro-geometries. Microscale fluid flow and heat transfer behavior differs greatly from that at macroscale. Many experimental and numerical investigations on fluid flow and heat transfer in micro-channels have been undertaken. It is known qualitatively that gaseous flow in a micro-channel is affected by the rarefaction (the slip on the surface), the surface roughness and the compressibility effects separately or simultaneously. The velocity slip and temperature jump are main effects of rarefaction.

Navier-stokes-based fluid dynamics solvers are often inaccurate when applied to MEMS. This inaccuracy stems from their calculation of molecular transport effects, such as viscous dissipation and thermal condition, from bulk flow quantities, such as mean velocity and temperature. This approximation of microscale phenomena with macroscale information fails as the characteristic length of the gaseous flow gradients (L) approaches the average distance traveled by molecules between collisions (the mean free path, λ) . The ratio of these quantities is known as Knudsen number, Kn, which defines flow characteristics when the flow dimensions approach the molecular mean free path.

$$Kn = \frac{\lambda}{L} \qquad (1)$$

Four different flow regims are defined based on the value of the Knudsen number, Kn [1]: Continuum flow for Kn ≤ 0.001; Slip flow regime for 0.001≤ Kn ≤ 0.1; transition regime for 0.1 ≤ Kn ≤ 10 and Free molecular regime for Kn ≥ 10. Kn is the ratio of gas mean free path, λ to characteristic dimension in the flow field, L and, it determines the degree of rarefaction and the degree of the validity of the continuum approach. As Kn increases, rarefaction effects become more important, and eventually the continuum approach breaks down. Palm [2], Sobhan and Garimella [3] and Obot [4] reviewed the experimental results in the existing literature for the convective heat transfer in microchannels. Morini [5] presents an excellent review of the experimental data for convective heat transfer in microchannels. Barron et al. [6,7] extended the Graetz problem to slip-flow and developed simplified relationships to describe the effect of slip-flow on the convection heat transfer coefficient. Ameel et al. [8] analytically treated the problem of laminar gas flow in micro tubes with a constant heat flux boundary condition at the wall assuming a slip flow hydrodynamic condition and a temperature jump thermal condition at the wall. They disclosed that the fully developed Nusselt number decreased with Knudsen number. Tunc and Bayazitoglu [9] studied steady laminar hydrodynamically developed flow in microtubes with uniform temperature and uniform heat flux boundary conditions using the integral technique. They included temperature jump condition at the wall and viscous heating within the medium. Orhan Aydin and MeteAvci [10] examanied laminar forced convective heat transfer of a Newtonian fluid in a microchannel between two parallel plate analytically. Sparrow et al. [11] investigated laminar heat transfer in microtubes under slip-flow conditions and Inman et al. [12] studied laminar slip-flow heat transfer in a parallel-plate channel with uniform wall heating and suggested a formula for calculation of Nusselt number in this condition.

In present study we performed computational study on heat transfer between parallel-plates under constant heat flux at wall in slip-flow regim.

2. Problem Description

We investigated hydrodynamically and thermally fully developed, steady, laminar, incompressible two-dimensional flow having constant properties. We consider the flow of gas through two parallel-plates of length *L* that are a distance *H* apart with constant heat flux at wall (fig. 1). Then Nusselt number can defied as:

$$Nu = \frac{hD_h}{k} = \frac{q_w\, 2H}{k(T_w - T_b)} \qquad (2)$$

Where h is the convection coefficient and D_h is the hydrodynamic diameter (=*2H*). And k is the thermal conductivity. T_b is the bulk temperature defined by:

$$T_b = \frac{\int \rho u T dA}{\int \rho u dA} \qquad (3)$$

As shown in figure, the velocity and temperature fields are uniform at inlet of the channel. And Reynolds number is defined as:

$$Re = \frac{\rho U_{in} D_h}{\mu} \qquad (4)$$

Where $D_h = 2H$, is hydraulic diameter of channel.

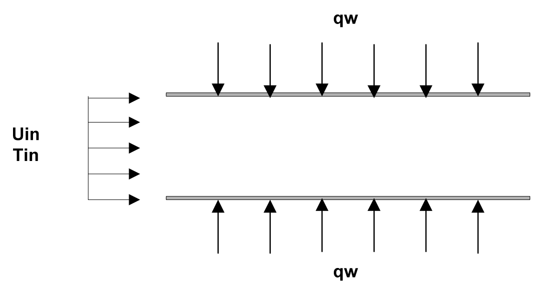

Fig. 1

Both walls are fixed. Nitrogen is selected as working fluid in this problem.

Table 1

Parameter boundary condition	value
L	10 µm
H	1 µm
Uin	7.2 m/s
Tin	300 k
Density , ρ	1.1381 kg/m^2
Dynamic viscosity, μ	17.86× $10^{-6}\ Ns/m^2$
Meen free path, λ	$6.044 \times 10^{-8}\,m$
σ_v, σ_T	1
Kn	0.06044

3. Analysis and numerical method

In peresent work, A new method for modeling of microflows is shown. Governing equations are developed by using perturbation expansions of velocity, pressure, temperature fields. Subsequently, different orders of equations in dependence of Knudsen number are obtained. This set of equations is discretized in two-dimensional state on a staggered grid using the finite volume method. In this study we use three-term perturbation expansions. Total algorithm of solution includes three steps and solved with the SIMPLE algorithm.

We solve Navier-stokes equations with slip and jump boundary conditions for velocity and temperature. Velocity slip is defined as: [13]

$$U_s - U_w = \frac{2-\sigma_v}{\sigma_v} Kn \frac{\partial U}{\partial \vec{n}} \quad (5)$$

Where U_s is the velocity of the gas at the wall and U_w is wall velocity. σ_v is the tangential momentum accommodation coefficient and the temperature jump is defined as: [13]

$$T_s - T_w = \left(\frac{2-\sigma_T}{\sigma_T}\right)\left(\frac{2\gamma}{\gamma+1}\right)\frac{Kn}{Pr}\frac{\partial T}{\partial \vec{n}} \quad (6)$$

Where T_s is the temperature of the gas at the wall, T_w is the wall temperature, and σ_T is the thermal accommodation coefficient. $\sigma_v$and σ_T are parameters that describe gas-surface interaction and are functions of the composition and temperature of gas, the gas velocity over the surface, and the solid surface temperature, chemical state and roughness. Particularly for most engineering applications they assume typical values near unity [13].

In this research we use three-therm expansions and reach to three order of equations O(1), O(Kn), O(Kn^2) and their boundary conditions. In fact the equation of O(1) are the no-slip navier-stokes equations. Also, the equations of (Kn) and O(Kn^2), govern required corrections due to the velocity-slip and temperature-jump. Total algorithm of solution includes three steps: the first step is the solution of the O(1) equations with the O(1) boundary conditions. The second step is solution of the O(Kn) equations with the O(Kn) boundary conditions. This step's boundary conditions are obtained by fitting the first step's fields on walls.The third step is solution of the O(Kn^2) equations with the O(Kn^2) boundary conditions. This step's boundary conditions are obtained by fitting the second step's fields on walls. Each part of this program, solve one order of the equations with the SIMPLE algorithm.

Momentum and energy equations are described as:

$$\rho\frac{D\vec{V}}{Dt} = -\nabla p + \mu\nabla^2\vec{V} \qquad (7)$$

$$\rho C_p\frac{DT}{Dt} = k\nabla^2 T \qquad (8)$$

First order boundary conditions are defined as:

$$U_0|_s = U_w \qquad (9)$$

$$T_0|_s = T_w \qquad (10)$$

Boundary conditions for second step are defined as:

$$U_1|_s = \frac{2-\sigma_v}{\sigma_v}L_c\left(\frac{\partial u_0}{\partial n}\right)|_s \qquad (11)$$

$$T_1|_s = \frac{2-\sigma_T}{\sigma_T}[\frac{2\gamma}{\gamma+1}]\frac{L_c}{Pr}\left(\frac{\partial T_0}{\partial n}\right)|_s \qquad (12)$$

Where L_c is characteristic length, u_0 and T_0 are velocity and temperature field for no-slip step. γ shows the specific heat ratio and Pr defines the Prandtl number. , u_1 and T_1 are solutions of the O(Kn) equations.

And the boundary conditions for O(Kn^2) are defied as:

$$u_2|_s = \frac{2-\sigma_v}{\sigma_v}[L_c\frac{\partial u_1}{\partial n} + \frac{{L_c}^2}{2}\frac{\partial^2 u_0}{\partial n^2}] \qquad (13)$$

$$T_2|_s = \frac{2-\sigma_T}{\sigma_T}[\frac{2\gamma}{\gamma+1}]\frac{1}{Pr}[L_c\frac{\partial T_1}{\partial n} + \frac{{L_c}^2}{2}\frac{\partial^2 T_0}{\partial n^2}] \qquad (14)$$

u_2 And T_2 are solutions of the O (Kn^2) equations. Finally by solving this steps and substituting the solutions in perturbation expansion for achieving the total temperature value:

$$U = U_0 + KnU_1 + Kn^2U_2 \qquad (15)$$

$$P = P_0 + KnP_1 + Kn^2P_2 \qquad (16)$$

$$T = T_0 + KnT_1 + Kn^2T_2 \qquad (17)$$

4. Results and Discussion

The computations were performed for a micr-channel with constant heat flux of $q'' = 10^3\, w\, m^{-2}k^{-1}$. Nitrogen with $\gamma = 1.4$, $\mu = 17{,}86 \times 10^{-6}$ pa s and $\lambda = 6.044 \times 10^{-8}$ was assumed for the working fluid. In the present work, effects of thermal creep and viscous dissipation are neglected and assumed that $\sigma_v = \sigma_T = 1$. In addition we considered that velocity and temperature profiles at inlet are uniform profiles. Grid size in this study is 30×300. We change Knudsen number by changing *H*. The numerical code is validated by results with the available numerical or analytical results. We compared our results with O.Aydin's[10] and Inman's [12] results and saw that our results have a good agreement with Inman's and Aydin's results.

For this case, first we solve momentum and energy equations with no-slip boundary conditions. Then by solving this equations with first and second order boundary conditions, And using perturbation expansion we can correct velocity and temperature fields. Solutions of all steps of this method are shown in figs(2). This figures are for a case that $= 0.5$. fig(2a), shows temperature distribution at $\frac{X}{L} = 0.5$ for first step of solution(no-slip). Fig(2b) and (2c) show distributions of T_1 and T_2 that are solutions for second and third steps of this method. finally by substitution

T_0, T_1 and T_2 in perturbation expansion we can achieve temperature filed at this position as shown in fig(2d). this figure shows difference between no-slip and slip solution clearly.

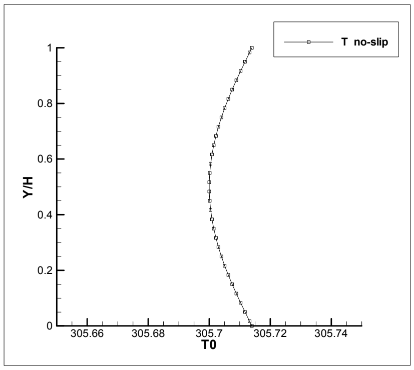

Fig (2a)

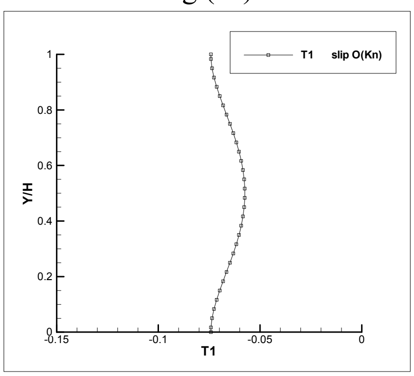

Fig (2b)

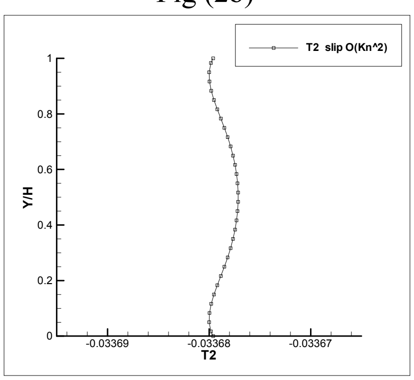

Fig (2c)

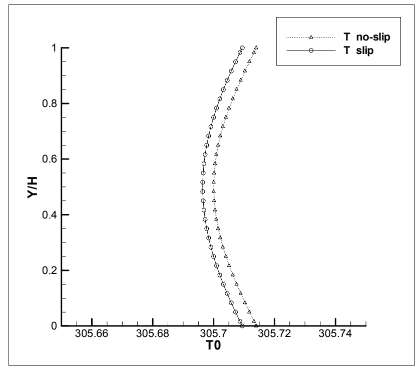

Fig (2d)

Also we presented temperature profiles along the channel length for three various sections in fig (3) by defining dimensionless temperature as:

$$\theta = \frac{(T - T_i)}{({}^{q_w H}/_{k})} \qquad (18)$$

Where T_i is fluid temperature at inlet.

Fig (3) shows how changes these profiles along the channel length.

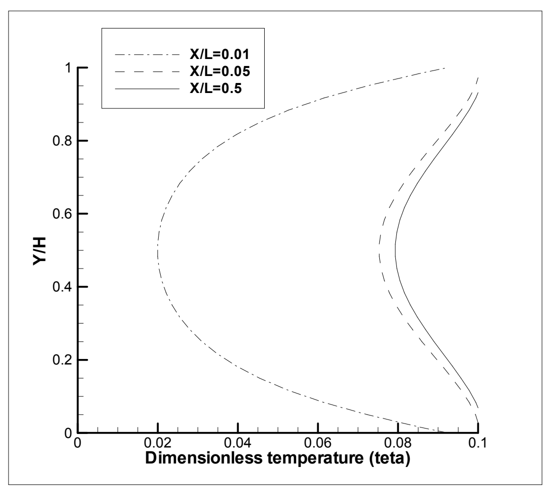

Fig (3) temperature profiles along the channel length for three various sections

Finally we compare Nusselt numbers that we obtain by this method against Inman and O.Aydin studies. This comparison is presented in table (2) and fig (4).

Table 2
The fully developed Nusselt number values, $Pr = 0.7$ for the CHF case

Kn	Present work Nu	Inman ref[12] Nu	O.Aydin and M.Avic ref[10] Nu
0.0	8.241	8.235	8.236
0.02	7.532	7.502	7.5
0.04	6.916	6.843	6.842
0.06	6.382	6.264	6.262
0.08	5.882	5.758	5.756
0.1	5.431	5.317	5.314

In low Knudsen numbers, we have good agreement between Nusselt numbers that obtain in our study and Nusselt numbers values that obtain by Inman and Aydin. but when Knudsen number increases,the deference between Nusselt number obtained in this paper and others increases, but in slip flow regim, for 0.001≤ Kn ≤ 0.1, this maximum deference is 2% approximately.

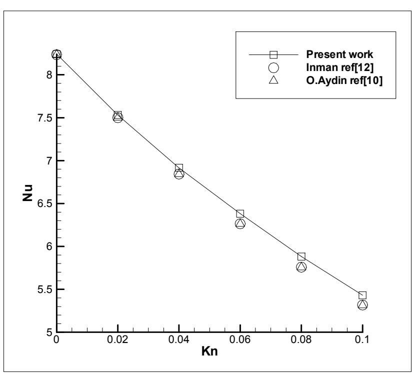

Fig (4)

5. Conclusions

Here we studied slip-flow heat transfer for laminar rarefied gas flow in microchannels between two parallel plates with uniform heat flux on walls. This study includes velocity slip and temperature jump at walls and neglected the effecs of thermal creep and viscous dissipation and compressibility. Finally the interactive effect of the Knudsen number on Nusselt number has been studied and found that the Nusselt number decreases when rarefaction increases.

Table 2
Symbols

Nomenclature	
A	Cross-sectiona area …………………… m^2
c_p	Specific heat at constant pressur…. $J/Kg.K$
c_v	Specific heat at constant volume .. $J/Kg.K$
D_h	Hydraulic diameter …………………..… m
h	Heat transfer coefficient ………...…. w/m^2k
H	Distance between two plates ………… m
k	Thermal conductivity ……………….. w/mk
Kn	Knudsen number, λ/L
L	Characteristic length …………………… m
Nu	Nusselt number, hD_h/k
Pr	Prandtl number, υ/α
q_w	Wall heat flux ……………….……… w/m^2
Re	Reynolds number, $\rho u D_h/\mu$
T	Temperature …………………..…………. K
T_w	Wall temperature ……………….……… K
T_s	bulk temperature of the fluid …..……… K
U	Axial velocity …………..…………… m/s
u_w	Wall velocity ……………………..…… m/s
u_s	Fluid particles velocity adjacent to the wall…. m/s
σ_v	Momentum accommodation coefficient
σ_T	Thermal accommodation coefficient

λ	Molecular mean free path …………	m
μ	Dynamic viscosity …………….	kg/ms
ν	Kinematic viscosity …………	m^2/s
ρ	Density ……………………….	kg/m^3
γ	Heat capacity ratio, c_p/c_v	
α	Thermal difiusivity …………	m^2/s

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