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Problem 1

Drazin & Reid 1.4

Hint: Be careful to distinguish between a particular *mode* being stable and the flow (all modes) being stable.

Problem 2

Drazin & Reid 1.10

Problem 3

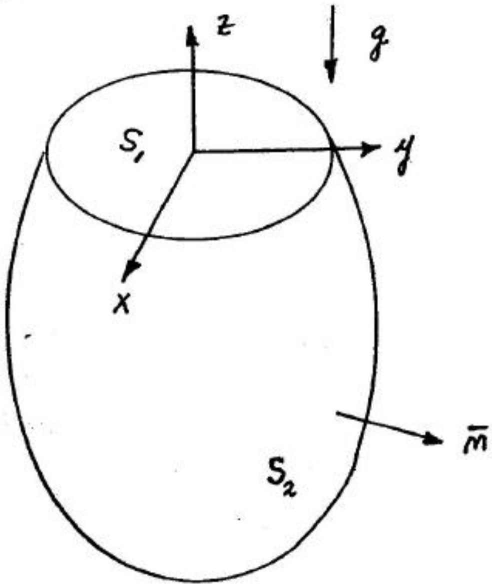
In our discussion of the Kelvin-Helmholtz instability, we have derived the boundary value problem for waves on an interface with both fluids of infinite horizontal and semi-infinite vertical extent. When the fluid in the lower region is bounded by a container, one need only modify our governing equations to require zero normal velocity at the boundary of the container. Let $\mathcal{D}$ be the region of the quiescent fluid in the container, S_1 be the the quiescent free surface, and S_2 be the container boundary with outward normal **n**. Defining $\eta(x, y, t)$ as the position of the free surface, linear free surface wave motion in the absence of surface tension is thus governed by the boundary-value problem

$$\nabla^2\phi = 0 \quad \text{in } \mathcal{D}$$

$$\phi_z = \eta_t \quad \text{on } S_1$$

$$\phi_t + g\eta = 0 \quad \text{on } S_1$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on } S_2$$



Consider planar standing waves in the rectangular channel sketched below. Thus we want to look for non-propagating waves where the potential function has the form $\phi(x,z,t) = Re\{F(x)G(z)e^{i\omega t}\}$, in which Re means "the real part of." Show first that the kinematic and dynamic free surface conditions can be written as

$$\phi_{tt} + g\phi_z = 0 \qquad \text{on } S_1$$

and then solve the problem (by separation of variables) to show that

$$\phi_{odd} = A \sin kx \cosh k(z+h) \cos \omega t \qquad \text{for} \quad k = \frac{(2n-1)\pi}{2b}$$

$$\phi_{even} = B \cos kx \cosh k(z+h) \cos \omega t \qquad \text{for} \quad k = \frac{n\pi}{b}$$

where $n = 1, 2, 3 \ldots$ and k is the wavenumber. Find the dispersion relation $\omega = \omega(g, k, h)$ and note that the sloshing frequency depends on both b and h. Compare your results for the first odd mode with the experiments performed in class in a plot of ω versus h.