Rotor Design and Analysis of Stall-regulated Horizontal Axis Wind Turbine

Xinzi Tang  
University of Central Lancashire, Preston, UK  
XTang4@uclan.ac.uk

Xiongwei Liu  
University of Central Lancashire, Preston, UK  
XLiu9@uclan.ac.uk

Ahmad Sedaghat  
University of Central Lancashire, Preston, UK  
ASedaghat@uclan.ac.uk

Lik-kwan Shark  
University of Central Lancashire, Preston, UK  
LShark@uclan.ac.uk

Abstract — Wind energy provides energy security at a time when decreasing global reserves of fossil fuel threatens the long-term sustainability of energy supply. The power performance of a wind turbine depends on the site’s wind speed distribution and the design characteristics of the wind turbine. Based on the relationship between the power performance and the rotor parameters, this paper addresses the design tip speed ratio, rated wind speed, rotor diameter, and blade geometry of a stall-regulated wind turbine. As a case study, a dedicated aerofoil is used for the design and analysis of a stall-regulated wind turbine blade based on a specific wind speed distribution.

Index Terms — Blade element momentum theory; Horizontal axis wind turbine; Power performance; Rotor design; Stall-regulated; Site-specific wind turbine

I. INTRODUCTION

Renewable energy is essential to the UK Government’s objectives to reduce carbon dioxide emissions by 60% by 2050 and to generate 15% of the UK’s electricity supply from renewable sources by 2020. As one of the main sources of renewable energy, wind energy is under development with an expectation of an additional 14GW energy capacity from onshore wind in the UK [1].

A wind turbine system extracts kinetic energy of the wind into mechanical power and converts it to electrical power. As power in the wind passes two different systems before it can be used, the output amount of power available is mainly affected by the efficiency of the turbine rotor and the efficiency of the mechanical and electrical systems. Rotor design is a complex problem and it is impossible to expect the maximum efficiency without an optimization process [2].

Based on the classical blade element momentum (BEM) theory, tremendous efforts have been put on wind turbine design and optimization, most of which take maximum annual energy production or minimum cost of energy as objectives [3]. The BEM theory has been widely applied as its comparable simplicity and verified having an acceptable accuracy before stall in many industrial cases. And the methods derived from the BEM theory have been converted into computational codes [4;5]. Providing a set of basic specifications and constrains, these methods are able to select a reasonable design over a range of options. As design parameters are given before the optimization procedure begins, the relationship between the power performance and the rotor parameters is not fully understood. Since the power performance of a wind turbine depends on the design characteristics but also the site specific wind resource, the rotor configuration should be carefully considered according to the site’s wind speed distribution.

Fix-pitch variable-speed (FPVS) stall-regulated horizontal axis wind turbines (HAWT) are recognized as a viable approach of power controlling in lieu of expensive pitch-controlled wind turbines. The rotor speed can be controlled within a certain range according to the maximum power coefficient law below rated wind speed. At above rated wind speed, the turbine limits its peak power by well designed blade stall. The design procedure and power performance estimation of a three-blade, upwind, stall-regulated HAWT is discussed in the following sections. The relationship between power performance and rotor specific parameters is analyzed and presented with a case study.

II. ROTOR DESIGN METHODOLOGY

In the design procedure, the main parameters such as aerfoil type, design tip speed ratio, rated wind speed, rotor diameter, should be considered first before conducting blade geometry optimization.

A. Aerofoil selection

Aerofoil for HAWT is often designed to be used at low attack angle, where the drag coefficient is usually much lower than the lift coefficient. A general aviation aerofoil shape is NACA series, and dedicated aerofoil shapes used in modern wind turbines are: S8 series developed by National Renewable Energy Laboratory (NREL) in USA, FFA-W series developed by FOI in Sweden, RisØ-A1 series developed by RisØ in Denmark, DU series developed by Delft University of Technology in Netherlands. It has been found in some applications that more than one aerofoil shape can be used for the wind turbine blade design, but there will be bending between these aerofoils which may add to uncertainties in the design process.

For a stall-regulated wind turbine, it is better to choose an aerofoil shape to make sure that stall occurs gently after the maximum lift-to-drag point. Design a wind turbine for a specific site should not only include an optimum geometry with the maximum power coefficient but also the detailed power coefficient curve which is a function of wind speeds or the tip speed ratio. With more accurate aerodynamic coefficients at high attack angles, the more accurate design and performance prediction can be obtained. But the
The aerodynamic coefficients of a rotating aerofoil are different from the ones of a linear moving aerofoil. The coefficients from wind tunnel testing are acceptably accurate in steady flow, but in stall conditions, these coefficients are always lack of accuracy or there is no coefficient measured at very high attack angles at all. The low maximum lift coefficient, 21% thickness-to-chord ratio NREL S809 aerofoil has been extensively used in HAWT, and the post-stall aerodynamic characteristics of S809 have been investigated and published. It is also used as a base case in this paper. The modified lift and drag coefficients for steady-state and post-stall performance prediction are shown in Fig.1. These coefficients were presented with consideration of 3-dimensional flow [6;7]. The lift coefficient for the S809 aerofoil increases to 1.32 at an attack angle of 15°, but then it decreases as attack angle increases. The drag coefficient increases after an attack angle of 9°, and the maximum lift-to-drag ratio occurs at 8°. As the wind speed increases to high wind speed, the blade comes into deep stall regime; therefore a constant power can be achieved with a well-designed blade aerofoil.

B. Design tip speed ratio

Given an aerofoil, the design tip speed ratio is the first parameter that used in a blade design procedure, which is generally taken as 6-8 in modern wind turbines. But the optimum value remains uncertain for different aerofoil shapes and blade numbers[8;9]. It was claimed that NACA 4415 has an optimum tip speed ratio of 8.5 while LS-1 has an optimum value of 10 in 3-blade turbines[8].

As a higher lift coefficient means a larger lift force, a higher drag coefficient means a larger drag force, a turbine with the aerofoil of a higher lift coefficient and a lower drag coefficient is expected to produce more power with better load conditions. The maximum lift-to-drag ratio should be used in the optimal design, and the attack angle at which the maximum lift-to-drag ratio occurs should be considered to be the optimal attack angle. This optimal attack angle, which is equal to the angle of relative wind minus twist angle and pitch angle at all sections when the blade geometry is optimal designed according to the BEM theory, should be used in the design to calculate ideal power coefficient.

The BEM theory divide a blade into several sections from root to tip and the total power coefficient is calculated by integrating the power coefficients at these sections, as described in [10]:

\[
C_p = \left( \frac{8}{\bar{\lambda}^2} \right) \int F \sin^2 \varphi \cos \varphi + \lambda, \sin \varphi \sin (\sin \varphi + \cos \varphi) (1)
\]

\[
\lambda = \left[ 1 - \left( \frac{C_d}{C_l} \right) \cot \varphi \right] \lambda_r
\]

Here, \(C_p\) is the power coefficient, \(C_l\) is the lift coefficient, \(C_d\) is the drag coefficient, \(\lambda\) is the tip speed ratio, \(\lambda_r\) is the speed ratio at hub (root), \(\lambda_r\) is the local speed ratio at position \(r/R\), \(\varphi\) is the angle of relative wind, and \(F\) is the tip loss factor. For a local loss calculation, it is described as Prandtl’s loss factor [10], which is a function of the local relative angle and the local tip speed ratio. Here, for the overall power coefficient calculation of an ideal pre-designed blade, it can be referred as follows, where \(Z\) is the number of blades, \(C_{pschmitz}\) is the theoretical coefficient including whirlpool losses[9]:

\[
C_p = \frac{C_{pschmitz}}{Z} \left( 1 - \frac{1}{\lambda} \right) \left( 1 - \frac{1.84}{\lambda_r} \right)
(2)
\]

From the above equations, it can be seen that there is a relationship between the ideal total power coefficient and different tip speed ratios. Given a maximum lift-to-drag ratio, the ideal total power coefficients versus different tip speed ratios can be obtained. From the modified lift and drag coefficients published, for S809 aerofoil, a maximum lift-to-drag ratio of 55.6 occurs at an attack angle of 8°; the ideal power coefficients versus different tip speed ratios are plotted in Fig. 2. It is shown that the optimum tip speed ratio for S809 is around 8.

C. Rated wind speed

The rated wind speed is the wind speed at which the wind turbine is generating its rated power. And the wind power is proportional to the cube of the wind speed; high wind speed
means high power can be produced. However, a higher rated wind speed is not always a good choice as the annual power output is also a function of the local wind speed distribution, which is generally described as Weibull distribution with a shape parameter and a scale parameter. The annual power output can be calculated as:

$$P_{annual} = 10^{-8} \cdot 8760 \cdot \frac{1}{2} \cdot \rho A C_{me} \int_{v_{in}}^{v_{out}} C_p(v) f_{weibull}(v) \, dv$$ (MWH)

(3)

Where, $\rho = 1.225 \text{ kg/m}^3$ is the air density at the sea level, $A_c$ is the wind turbine rotor area, in $\text{m}^2$, $C_{me}$ is the mechanical and electrical system efficiency, and $C_p$ is the aerodynamic power coefficient of the rotor, which is basically determined by the rotor design and is a dimensionless function of wind speed (or tip speed ratio).

For a stall-regulated wind turbine, there is not a simple way to express the real power output exactly in a mathematical expression at above rated speed. But for estimation, between the cut-in wind speed and the rated wind speed, the rotor should work at its maximum efficiency with an optimum control strategy, and between the rated wind speed and the cut-out wind speed, the rotor is expected to produce a constant rated power. And the Weibull parameters vary with local wind conditions, the shape parameter here is taken as 2 as an example here. With a shape parameter of 2 Weibull distribution is also known as Rayleigh distribution, then (3) is converted into:

$$P_{annual} = 10^{-8} \cdot 8760 \cdot \frac{1}{2} \cdot \rho A C_{me} \int_{v_{in}}^{v_{out}} C_p(v) \frac{\pi}{2} \frac{v}{V_{mean}} \exp\left(-\frac{\pi^2}{4 \frac{v^2}{V_{mean}^2}}\right) \, dv$$

$$+ 10^{-8} \cdot 8760 \cdot C_{me} \cdot P_{rated} \int_{v_{in}}^{v_{out}} \frac{\pi}{2} \frac{v}{V_{mean}} \exp\left(-\frac{\pi^2}{4 \frac{v^2}{V_{mean}^2}}\right) \, dv$$ (MWH)

(4)

Based on the above analysis, it is obviously that choosing an appropriate rated wind speed based on the local wind distribution is critical for the annual power output. An important parameter to describe wind resource used is annual mean wind speed. It seems that given a high mean wind speed $V_{mean}$, it is possible to produce more power with a high rated wind speed. With a low mean wind speed resource, it is likely to produce more power with a low rated wind speed. But it is shown in the following analysis that it is not the mean wind speed should be selected as rated wind speed.

As the wind power is proportional to the cube of the wind speed, let’s define the annual mean cubic wind speed:

$$V_{mean\cubic} = \left(\int_{v_{in}}^{v_{out}} v f_{weibull}(v) \, dv\right)^{\frac{1}{3}}$$ (m/s)

(5)

and defining the annual mean wind speed as:

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then we can establish the relationship between the annual mean cubic wind speed and the mean wind speed.

By defining a dimensionless factor $c$:

$$C = \frac{V_{mean\cubic}}{V_{mean}}$$

(7)

With a shape parameter of 2, the annual mean cubic wind speed is found about 1.24 times as mean wind speed:

$$C = \sqrt[3]{\int_{v_{in}}^{v_{out}} v f_{weibull}(v) \, dv}$$

$$= \frac{V_{mean\cubic}}{V_{mean}} = 1.24$$

(8)

A Matlab program has been developed to find the relationship between the annual power output and the rated wind speed based on the ideal power output from (4). It is found that an optimum rated wind speed depends on the mean wind speed. With $C_p=0.4$, $C_{me}=0.82$, as shown in Fig.3, for a 10kW wind turbine, with a mean wind speed higher than 3 m/s, the annual power output is lower with a higher factor $k$, where $k$ is defined as $k=V_{rated} / V_{mean}$. But it is also worth emphasizing that a lower rated wind speed means a larger generator, which causes an increase in cost.

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aerodynamic shape with nonlinear chord and twist distribution, which can be obtained based on the BEM theory with respect to a certain aerofoil type.

It can be seen from (1) that, there is a relationship between these design parameters and the maximum power coefficient. If the main part of the equation is at its maximum, as shown in (10), the total power coefficient is maximized.

\[
F \sin^2 \varphi (\cos \varphi - \lambda, \sin \varphi (\sin \varphi + \lambda, \cos \varphi) \lambda^2 \left[ 1 - (C_d / C_r) \cot \varphi \right] \rightarrow \text{Max}
\]

Ignoring the drag-to-lift coefficient ratio and setting the partial derivative of the main part to zero, the optimum design equation for any kind of aerofoil can be obtained [10]:

\[
\varphi = \left( \frac{2}{3} \right) \tan^{-1} \left( \frac{1}{\lambda t} \right)
\]

(11)

\[
C_r = \frac{8n}{ZC} \left( 1 - \cos \varphi_r \right)
\]

(12)

The chord and twist distributions from the above equations are just an initial design values and iterations are followed normally. Some modifications of the twist angle and chord length distributions may be conducted to decrease the drag and thrust forces to the rotor at high winds. However they should be close to the theoretical distributions so as to make sure the maximum power coefficient and low start-up characteristics can be achieved.

### III. Case Study

A case study has been conducted with a 10kW fix-pitch variable-speed stall-regulated horizontal axis wind turbine, and with the specifications of the rotor listed in Table I. The local wind speed distribution is set to fit Rayleigh distribution with the UK mean wind speed of 5m/s at the rotor centre height, and a rated wind speed of 8m/s is selected. The blade has an aerofoil of S809 with a thickness of 21% chord length along the blade except for the transitional region from the root to the first section of S809 aerofoil, and the design tip speed ratio is set at 8.

### IV. Results and Discussion

The blade geometry is divided into 20 sections, and the initial distributions of the twist and the chord are shown in Fig.4 and Fig.5. The Power coefficients of all the sections are plotted in Fig.6. The power coefficients versus rotor rotational speed with different tip speed ratio are plotted in Fig.7.

<table>
<thead>
<tr>
<th>ROTOR SPECIFICATIONS</th>
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<tbody>
<tr>
<td>Rated power w</td>
</tr>
<tr>
<td>Rated wind speed m/s</td>
</tr>
<tr>
<td>Aerodynamic power coefficient</td>
</tr>
<tr>
<td>Number of blades</td>
</tr>
<tr>
<td>Design tip speed ratio</td>
</tr>
<tr>
<td>Mechanical and electrical efficiency</td>
</tr>
<tr>
<td>Radius of the rotor m</td>
</tr>
<tr>
<td>Design attack angle degree</td>
</tr>
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Fig.4 and Fig.5 demonstrate that the chord length is larger at the inner sections of the blade (closer to the root) and smaller at the outer sections of the blade. The twists of the sections close to the root are larger than those close to the tip. The root sections are set to high twist angles, which contribute to the lower start up performance as the large twist angle makes the root sections to have an appropriate attack angle at a low start-up wind but are more likely to stall at high wind speed. And the twist angle at the tip section is about –1.69 degree which also makes the tip sections more likely to stall at high wind speeds but will contribute at low wind speed. Therefore modifications should be balanced between the two aspects. Any modification should make the real attack angle to approximate the optimum attack angle and make sure the lift force is not decreased sharply.

Fig.6 reveals that the outer sections produce most of the power at the rated wind speed. Therefore, any modification for manufacturing considerations on the chord length and twist distributions of these sections should be limited close to the initial optimal design to ensure rated power.
It is suggested in Fig.7 that, to obtain a maximum power coefficient before the rated wind speed, the rotor rotational speed should be variable according to the maximum power coefficient point. A detailed control strategy can be implied, which is out of the theme of this paper.

V. CONCLUSION

Various rotor parameters were investigated in this paper, and the rotor design and analysis were conducted based on a case of S809 aerofoil. The outcome demonstrates that:

1) A set of optimum design parameters should be considered to achieve the maximum annual power output with a local wind speed distribution.
2) For a stall-regulated wind turbine, it is appropriate to choose an aerofoil shape to make sure gentle stall occurs after the maximum lift-to-drag point.
3) The design tip speed ratio should be selected according to the aerodynamic characteristics of the aerofoil.
4) The rated wind speed should be selected based on the local site’s mean wind speed.
5) The chord and twist distribution of the blade can be obtained based on BEM theory, and any modifications on chord and twist distributions should approximate the optimum distribution.
6) A detailed power coefficient curve can be predicted based on lift and drag coefficients considering 3-dimensional flow for stall-regulated wind turbines. According to the power coefficient curve, the maximum power coefficient control strategy can be applied.

ACKNOWLEDGEMENTS

The first author gratefully acknowledges the financial support of University of Central Lancashire Addison Studentship for this study.

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